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Enhanced NSGA-II Algorithm for Solving Multi-objective Optimization Problems Using PBX Crossover and POM Mutation.

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Abstract— **Non-Dominated Sorting Genetic Algorithm (NSGA-II) is an algorithm given to solve the Multi-Objective Optimization (MOO) problems. NSGA-II is one of the most widely used algorithms for solving MOO problems. The present work proposed as advancement to the existing NSGA-II. It this method, combination of crossover and mutation operators is used other than that in the original NSGA-II, and the results are compared to see if which works better.**

Keywords— **NSGA-II, Parent-centric Blend Crossover, Power Mutation.**

I. INTRODUCTION

The use of technology has increased rapidly over the years and so has increased the usage of software. This makes it important to maintain the quality of the software. Software testing is the most significant analytic quality assurance measure consuming at least 50% of software development cost [3]. The automation process of test data generation is a way that will reduce the time taken up by this task. Genetic Algorithm (GA) is used for this purpose.

A number of multi-objective evolutionary algorithms have been suggested earlier. Non-Dominated Sorting Genetic Algorithm (NSGA-II) is an algorithm given to solve the Multi-Objective Optimization (MOO) problems. It was proposed by Deb et.al in 2002 [4], advancing on the concept given by Goldberg 1989 [1]. NSGA-II is one of the most widely used algorithms for solving MOO problems. Rest of the paper is organized as follows: NSGA-II in second section, Proposed Methodology in third section.

II. NSGA-II

A. NSGA-II

A number of multi-objective evolutionary algorithms have been suggested earlier such as Pareto Archived Evolution Strategy (PAES) & Strength Pareto Evolutionary Algorithm (SPEA), Non-Dominated Sorting GA (NSGA) etc. The Non-dominated Sorting Genetic Algorithm [4] NSGA-II uses a faster sorting procedure, an elitism preserving approach and a parameter less niching operator. The working is given as follows [6]:

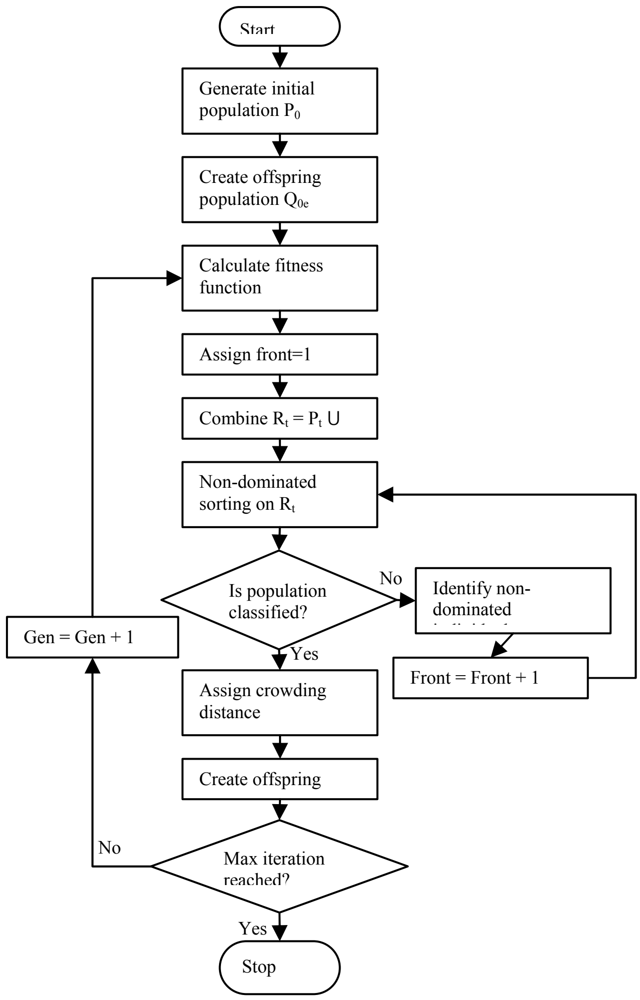


Fig. 1 WORKING OF NSGA II

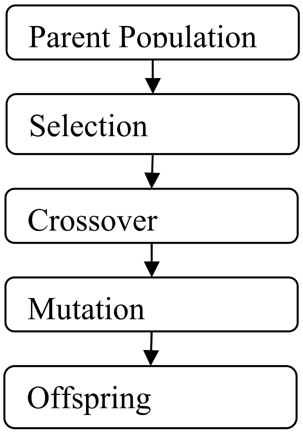


FIG. 2. CREATING OFFSPRING

B. *Selection [7]*

In original NSGA II *Binary tournament selection (BTS)* is used, where tournament is played between two solutions and better is selected and placed in mating pool. Two other solutions are again taken and another slot in mating pool is filled. It is carried in such a way that every solution can be made to participate in exactly 2 tournaments.

C. *Crossover [8]*

In NSGA II Simulated Binary Crossover (SBX) is used, which works with two parents solutions and create two offspring. The following step-by-step procedure is followed:

Step 1: Choose a random number $u_i \in [0,1]$,
Step 2: Calculate using equation 1,
Step 3: Compute offspring using equation 2.

The mathematical formulation can be given as follows:

$$\beta_{q_i} = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}} & \text{if } u_i \le 0.5; \\ \left(\frac{1}{2(1-u_i)}\right)^{\frac{1}{\eta_c+1}} & \text{otherwise.} \end{cases} \quad (1)$$

$$x_i^{(1,t+1)} = 0.5\left[\left(1+\beta_{q_i}\right)x_i^{(1,t)} + \left(1-\beta_{q_i}\right)x_i^{(2,t)}\right],$$
$$x_i^{(2,t+1)} = 0.5\left[\left(1-\beta_{q_i}\right)x_i^{(1,t)} + \left(1+\beta_{q_i}\right)x_i^{(2,t)}\right]. \quad (2)$$

Here,
u_i : Random number such that $u_i \in [0, 1]$,
η_c : Distribution index (Non-negative real number),
$x_i^{(1,t)}$ & $x_i^{(2,t)}$: Parent solutions,
$x_i^{(1,t+1)}$ & $x_i^{(2,t+1)}$: Offspring solutions.

D. *Mutation [8]*

In NSGA II Polynomial mutation is used, which mutates each solution separately, i.e. one parent solution gives one offspring after being mutated. The mathematical formulation can be given as:

$$y_i^{(1,t+1)} = x_i^{(1,t+1)} + (x_i^{(U)} - x_i^{(L)})\bar{\delta}_i , \quad (3)$$

Where,

$$\bar{\delta}_i = \begin{cases} (2r_i)^{1/(\eta_m+1)} - 1 & \text{if } r_i < 0.5, \\ 1 - [2(1-r_i)]^{1/(\eta_m+1)}, & \text{if } r_i \ge 0.5. \end{cases} \quad (4)$$

Here,
r_i : Random number such that $u_i \in [0, 1]$,
η_m : Distribution index (Non-negative real number),
$x_i^{(1,t+1)}$: Parent solution,
$x_i^{(U)}$: Upper bound of parent solution,
$x_i^{(L)}$: Lower bound of parent solution,
$y_i^{(t+1)}$: Offspring solution.

E. *Crowded Tournament Selection [6]*

To get an estimation of the density of solutions close to a particular solution i in the population, we take the average of the two solutions on the either side of solution i along each of the objective. This quantity d_i is the Crowding Distance. The following algorithm is used to calculate the crowding distance of each point in the set $\mathcal{F}$.

Assignment procedure: Crowding-sort($\mathcal{F}, <_c$)

Step 1: Call the number of solutions in $\mathcal{F}$ as $l = |\mathcal{F}|$. For each i in the set, first assign $d_i = 0$.

Step 2: For each objective function $m = 1, 2, \ldots, M$, sort the set in worse order of f_m. Find sorted indices vector $I_m = \text{sort}(f_m, >)$.

Step 3: For $m = 1, \ldots, M$, assign a large distance to the edge solutions, $d_{I_1^m} = d_{I_l^m} = \infty$, and for all other solutions $j = 2$ to $(l-1)$, assign:

$$d_{I_j^m} = d_{I_j^m} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j-1}^m)}}{f_m^{max} - f_m^{min}} \quad (5)$$

III. PROPOSED METHODOLOGY

This section gives an overview of the proposed methodology. In the proposed work we replace the crossover and mutation used in original NSGA-II, Simulated Binary Crossover (SBX) and Polynomial Mutation (PM) are replaced by Parent-Centric Blend Crossover (PBX) [5] and Power Mutation (POM). Also the Binary tournament selection procedure is excluded in a hope to get better result.

A. *Parent-Centric Blend Crossover (PBX)*

PBX or PBX-α is an extension to BLX-α [3] and is described as follows: Let us assume that $X = (x_1 \ldots x_n)$ and $Y = (y_1 \ldots y_n)$ $(x_i, y_i \in [a_i, b_i] \subset \Re, i = 1 \ldots n)$ are two real-coded variables that have been selected to apply the crossover operator to them. PBX- α generates (randomly) one of these two possible offspring:

$$Z_1 = (z_1^1 \ldots z_n^1) \text{ or } Z_2 = (z_1^2 \ldots z_n^2), \quad (6)$$

Where z_i^1 is a randomly (uniformly) chosen number from the interval $[l_i^1, u_i^1]$ with

$$l_i^1 = \max\{a_i, x_i - I * \alpha\} \text{ And } u_i^1 = \min\{b_i, x_i + I * \alpha\} \quad (7)$$

And z_i^2 is chosen from $[l_i^2, u_i^2]$ with

$$l_i^2 = \max\{a_i, y_i - I \text{ And } u_i^2 = \min\{b_i, y_i + I * \alpha\} \quad (8)$$

Where $I = |x_i - y_i|$.

B. *Power Mutation (POM)*

In [2] a new mutation operator called Power Mutation (POM) was introduced for real coded genetic algorithms.

The following formula is used to form the mutated solution:

$$\acute{x} = \begin{cases} x - s(x - l), & \text{if } t < \alpha \\ x + s(u - x), & \text{if } t \ge \alpha \end{cases} \quad (9)$$

Where l and u are lower and upper bounds of the decision variable and α is a uniformly distributed random number between 0 and 1, and s is a random number.

C. *Problem Statement*

In [9] Schaffer gave two-objective problems:

$$SCH1 = \begin{cases} \text{Minimize } f_1(x) = x^2 \\ \text{Minimize } f_2(x) = (x-2)^2 \\ -A \le x \le A. \end{cases} \quad (10)$$

Schaffer's second function, SCH2,

$$SCH2 = \begin{cases} \text{minimize} \quad f_1(x) = \begin{cases} -x & \text{if } x \le 1, \\ x-2 & \text{if } 1 < x \le 3, \\ 4-x & \text{if } 3 < x \le 4, \\ x-4 & \text{if } x > 4, \end{cases} \\ \text{minimize} \quad f_2(x) = (x-5)^2 \\ -5 \le x \le 10 \end{cases} \quad (11)$$

IV. Parameter Setup For Experiment

In this paper parameter settings are as; Population size = 20, Maximum number of cycles (MCN) = 100. $\eta_c = 20$, $\eta_m = 20$, $\alpha = 0.7$, $s = 0.3$.
(X-axis represents f1 & y-axis represents f2.)

TABLE 1
TIME TAKEN IN MILLISECONDS

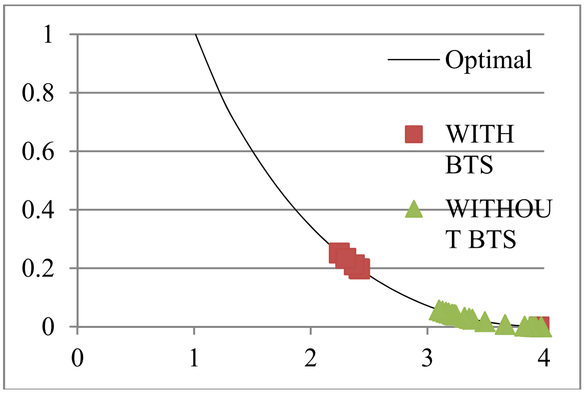
Problem	Combination	With BTS	Without BTS
SCH1	SBX-PM	145	140
	SBX-POM	136	138
	PBX-PM	140	143
	PBX-POM	**133**	**140**
SCH2	SBX-PM	130	174
	SBX-POM	**128**	**140**
	PBX-PM	132	168
	PBX-POM	125	150

TABLE 2
GENERATION 100 SCH1 SBX-PM

With BTS		Without BTS	
F1	F2	F1	F2
2.371634	0.21159	3.975206	3.85E-05
2.298813	0.234078	3.975034	3.91E-05
2.298813	0.234078	3.926204	0.000344
2.242577	0.252482	3.906979	0.000547
2.371634	0.21159	3.881965	0.000884
2.242577	0.252482	3.833179	0.001777
2.242577	0.252482	3.665175	0.007316
2.412973	0.199474	3.492437	0.017212
2.242577	0.252482	3.384913	0.02566
2.371634	0.21159	3.358004	0.028061
2.298813	0.234078	3.317818	0.031867
3.956474	0.000119	3.242758	0.039694
2.412973	0.199474	3.240626	0.039931
2.298813	0.234078	3.234235	0.040643
2.419103	0.197717	3.225016	0.041684
2.412973	0.199474	3.211284	0.043262
2.242577	0.252482	3.182015	0.046734
2.371634	0.21159	3.158029	0.049691
2.371634	0.21159	3.129007	0.053407
2.298813	0.234078	3.099577	0.057331

TABLE 3
GENERATION 100 SCH1 SBX-POM

With BTS		Without BTS	
F1	F2	F1	F2
11.68477	2.011575	3.940092	0.000226
11.68477	2.011575	3.620334	0.009464
11.68477	2.011575	3.602233	0.010413
11.68477	2.011575	3.575823	0.011884
11.68477	2.011575	3.572489	0.012077
11.68477	2.011575	3.129007	0.053407
11.68477	2.011575	2.995855	0.07244
11.68477	2.011575	2.794075	0.10788
11.68477	2.011575	2.767892	0.113099
11.68477	2.011575	2.686809	0.130214
11.68477	2.011575	2.655617	0.137191
11.68477	2.011575	2.581277	0.154736
11.68477	2.011575	1.891265	0.390334
11.68477	2.011575	1.834229	0.416881
11.68477	2.011575	1.826128	0.420756
11.68477	2.011575	1.781243	0.442714
11.68477	2.011575	1.740478	0.463391
11.68477	2.011575	1.236766	0.788366
11.68477	2.011575	0.626938	1.459761
11.68477	2.011575	4.023694	3.50E-05



GRAPH 1 SCH1- SBX-PM

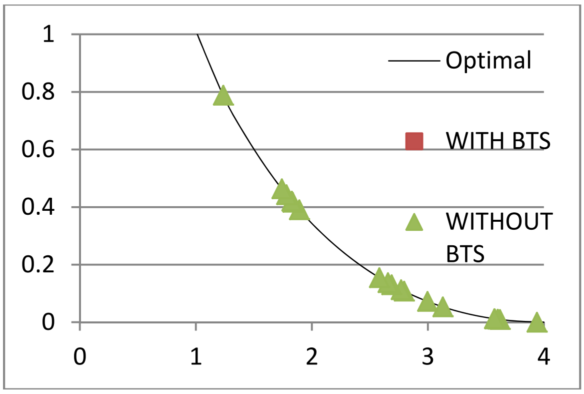
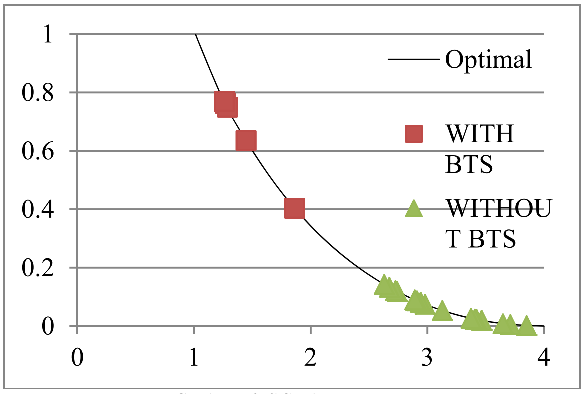


TABLE 8

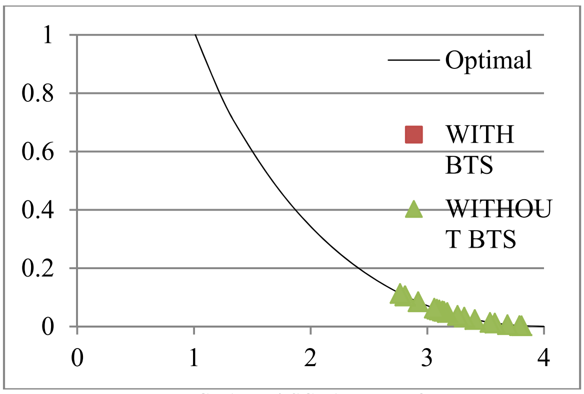
GRAPH 2 SCH1- SBX-POM



GRAPH 3 SCH1- PBX-PM

TABLE 4
GENERATION 100 SCH1 PBX-PM

With BTS		Without BTS	
F1	F2	F1	F2
1.445665	0.636233	3.851107	0.001412
1.269581	0.762554	3.711798	0.005387
1.25942	0.770465	3.64839	0.008086
1.286101	0.749845	3.466731	0.019067
1.445665	0.636233	3.464534	0.019231
1.25942	0.770465	3.422077	0.022534
1.286101	0.749845	3.406209	0.023842
1.861604	0.40398	3.373661	0.02665
1.25942	0.770465	3.129007	0.053407
1.25942	0.770465	2.979971	0.074934
1.269581	0.762554	2.944412	0.080696
1.269581	0.762554	2.942353	0.081038
1.861604	0.40398	2.912155	0.08614
1.25942	0.770465	2.897923	0.088608
1.862868	0.403391	2.891989	0.089649
1.861604	0.40398	2.891989	0.089649
1.286101	0.749845	2.742219	0.11836
1.25942	0.770465	2.726811	0.121588
1.445665	0.636233	2.674732	0.132889
1.25942	0.770465	2.630781	0.142908



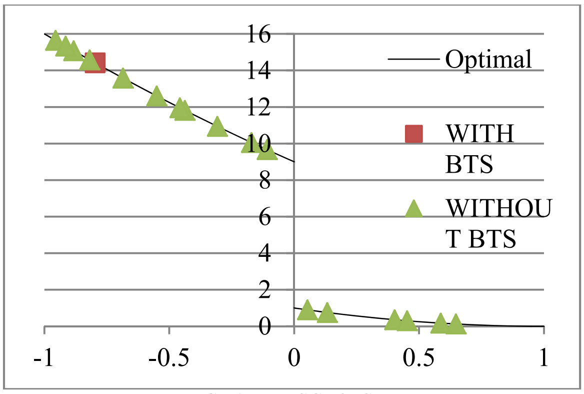
GRAPH 4 SCH1- PBX-POM

TABLE 5
GENERATION 100 SCH1 PBX-POM

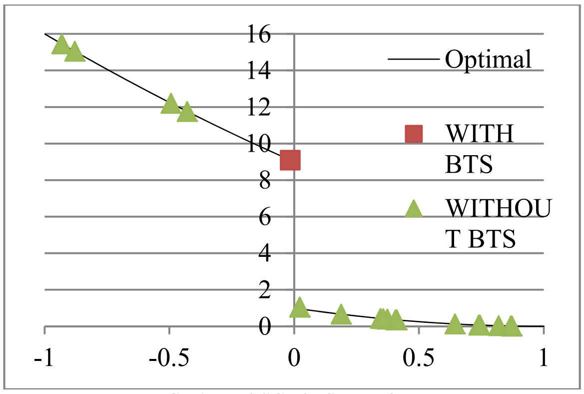
With BTS		Without BTS	
F1	F2	F1	F2
10.21206	1.429535	3.807913	0.002363
10.21206	1.429535	3.789182	0.002853
10.21206	1.429535	3.686606	0.006391
10.21206	1.429535	3.578379	0.011737
10.21206	1.429535	3.537751	0.014187
10.21206	1.429535	3.406908	0.023783
10.21206	1.429535	3.315772	0.032067
10.21206	1.429535	3.258854	0.037935
10.21206	1.429535	3.170908	0.048091
10.21206	1.429535	3.143091	0.051585
10.21206	1.429535	3.129007	0.053407
10.21206	1.429535	3.102989	0.056868
10.21206	1.429535	3.081838	0.059772
10.21206	1.429535	3.059641	0.062909
10.21206	1.429535	2.921935	0.084468
10.21206	1.429535	2.92178	0.084494
10.21206	1.429535	2.810019	0.104774
10.21206	1.429535	2.809372	0.104899
10.21206	1.429535	2.807722	0.105219
10.21206	1.429535	2.767019	0.113275

TABLE 6
GENERATION 100 SCH2 SBX-PM

With BTS		Without BTS	
F1	F2	F1	F2
-0.79742	14.42039	1.014899	0.000222
-0.79742	14.42039	-0.99907	16.00746
-0.79742	14.42039	0.647678	0.124131
-0.79742	14.42039	0.132641	0.752311
-0.79742	14.42039	0.40235	0.357186
-0.79742	14.42039	-0.30783	10.94175
-0.79742	14.42039	-0.55041	12.60541
-0.79742	14.42039	0.586655	0.170854
-0.79742	14.42039	-0.17118	10.05636
-0.79742	14.42039	0.452122	0.30017
-0.79742	14.42039	-0.81863	14.58192
-0.79742	14.42039	-0.43778	11.81835
-0.79742	14.42039	-0.6847	13.57698
-0.79742	14.42039	-0.68547	13.58268
-0.79742	14.42039	0.053068	0.896681
-0.79742	14.42039	-0.45738	11.95347
-0.79742	14.42039	-0.88326	15.07968
-0.79742	14.42039	-0.95544	15.64547
-0.79742	14.42039	-0.10872	9.664129
-0.79742	14.42039	-0.91519	15.3287



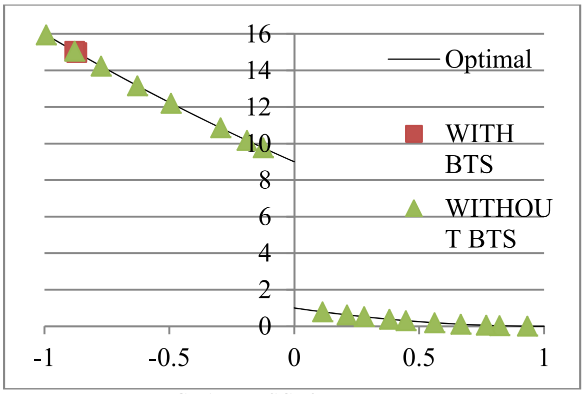
GRAPH 5 SCH2- SBX-PM



GRAPH 6 SCH2- SBX-POM

TABLE 7
GENERATION 100 SCH2 SBX-POM

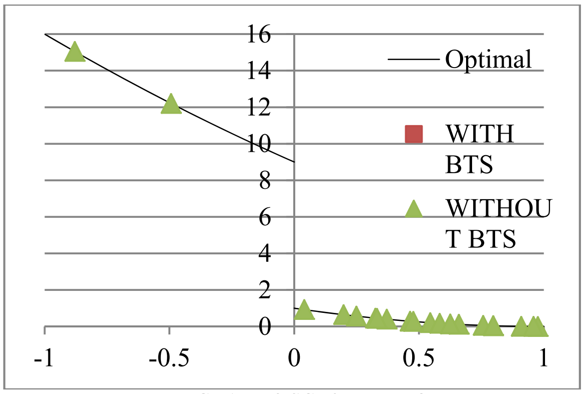
With BTS		Without BTS	
F1	F2	F1	F2
-0.01596	9.095996	1.049245362	0.00242511
-0.01596	9.095996	-0.93086437	15.4516947
-0.01596	9.095996	0.021972719	1.04442824
-0.01596	9.095996	-0.4934	12.2038436
-0.01596	9.095996	-0.8793	15.0489685
-0.01596	9.095996	-0.42904792	11.7583696
-0.01596	9.095996	0.6439	0.12680721
-0.01596	9.095996	0.188317144	0.65882906
-0.01596	9.095996	0.410889458	0.34705123
-0.01596	9.095996	0.871277625	0.01656945
-0.01596	9.095996	0.345635671	0.42819267
-0.01596	9.095996	0.021972719	1.04442824
-0.01596	9.095996	0.818777344	0.03284165
-0.01596	9.095996	0.739416458	0.06790378
-0.01596	9.095996	0.744361892	0.06535084
-0.01596	9.095996	0.867520091	0.01755093
-0.01596	9.095996	0.373213452	0.39286138
-0.01596	9.095996	0.405601506	0.35330957
-0.01596	9.095996	0.356479324	0.41411886
-0.01596	9.095996	0.871277625	0.01656945



GRAPH 7 SCH2- PBX-PM

TABLE 8
GENERATION 100 SCH2 PBX-PM

With BTS		Without BTS	
F1	F2	F1	F2
-0.8793	15.04897	1.000255747	6.54E-08
-0.46033	11.97385	-0.99339339	15.9471908
-0.76474	14.17325	1.000255747	6.54E-08
-0.77999	14.28836	-0.4934	12.2038436
-0.46033	11.97385	-0.62773406	13.1604544
-0.87073	14.98255	-0.29502739	10.8572055
-0.87073	14.98255	-0.77339606	14.2385178
-0.87073	14.98255	0.56177694	0.19203945
-0.87073	14.98255	0.666815309	0.11101204
-0.87073	14.98255	-0.8793	15.0489685
-0.46033	11.97385	0.44704394	0.3057604
-0.93766	15.50515	0.933359747	0.00444092
-0.87073	14.98255	0.279678309	0.51886334
-0.76474	14.17325	0.821712747	0.03178634
-0.8793	15.04897	0.380968309	0.38320023
-0.87073	14.98255	-0.18899939	10.1697171
-0.77999	14.28836	0.211168747	0.62225475
-0.87073	14.98255	0.768172747	0.05374388
-0.8793	15.04897	-0.12443439	9.76209029
-0.76474	14.17325	0.112873605	0.78699324



GRAPH 8 SCH2- PBX-POM

TABLE 9
GENERATION 100 SCH2 PBX-POM

With BTS		Without BTS	
F1	F2	F1	F2
2.344342	1.807256	0.97586227	0.00058263
2.344342	1.807256	-0.8793	15.0489685
2.344342	1.807256	-0.4934	12.2038436
2.344342	1.807256	0.039950341	0.92169535
2.344342	1.807256	0.909492466	0.00819161
2.344342	1.807256	0.197952779	0.64327974
2.344342	1.807256	0.325050591	0.4555567
2.344342	1.807256	0.756685924	0.05920174
2.344342	1.807256	0.370771194	0.39592889
2.344342	1.807256	0.248646599	0.56453193
2.344342	1.807256	0.659720821	0.11578992
2.344342	1.807256	0.797267823	0.04110034
2.344342	1.807256	0.464027764	0.28726624
2.344342	1.807256	0.545032875	0.20699508
2.344342	1.807256	0.476885137	0.27364916
2.344342	1.807256	0.958547937	0.00171827
2.344342	1.807256	0.582597783	0.17422461
2.344342	1.807256	0.625206869	0.14046989
2.344342	1.807256	0.331361833	0.447077
2.344342	1.807256	0.797267823	0.04110034

V. EXPERIMENT RESULT

Analysing the above graph we see that the optimization done without BTS clearly outperforms optimization done with BTS if spread of solutions in Pareto-Optimal set is considered. Moreover with SCH1 Combination of SBX and POM gives better result and for SCH2 Combination of PBX and PM gives better result than others.

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